REVIEWER'S REPORT

on PhD Thesis of Zhanuzakova Dinara Taupikhovna intituled "Methods of approximate solution of direct and inverse ptoblems of filtration theory"

Specialty: 6D060100 – Mathematics

The Thesis, proposed by applicant Zhanuzakova Dinara, is devoted to the study of direct and inverse problems for some mathematical models arising in filtration theory. In this work, the mathematical model of the theory of filtration taking into account the phase transition, the initial-boundary problem for the pseudo-parabolic equation, the inverse problem for the parabolic-type equation, and the reaction-diffusion problem are considered.

The solution of many important applied problems which arise in the study of liquid filtration processes in fractured porous media, the movement of groundwater with a free surface in multilayered media, the transfer of moisture, heat and salt in porous media is connected with the need to study initial-boundary value problems for parabolic and pseudoparabolic equations.

The presented work of the applicant Zhanuzakova D.T. is devoted to the study of the solvability of initial-boundary value problems in the theory of filtration.

The objects of the present dissertation are:

-To study the correctness of the mathematical model of the Stefan-type problem, the existence and uniqueness of the solution; the convergence to the limit in terms of relaxation time.

- Study the unique solvability of the initial-boundary problems for the quasi-linear pseudoparabolic equation with nonlinear boundary condition.

-She prove a theorem on existence and uniqueness of a weak generalized solution of problems, a blow-up result for the solution of a quasilinear pseudoparabolic equation with a nonlinear boundary condition, and study the asymptotic stability of solution in time.

- To find a weak solution of the inverse problem for the quasilinear parabolic equation, to study the stability of the solution, and also to prove that the solution blows-up in finite time.

- To show the existence of periodical stationary solutions of a competition system with nonlinear cross-diffusion (one-phase and two-phase patterns). Find domains in parameter space where these solutions exist.

The given Thesis consists of an introduction, five sections and a conclusion.

In the introduction of the presented Thesis first the relevance, actuality and novelty of results mentioned. After it the main objects are formulated. The summary of Thesis is presented as well.

In first section she presents the main functional spaces, known inequalities and definitions, lemmas, theorems that are used in the sequel.

The second section of the dissertation is devoted to study of the mathematical model from theory of filtration, which takes into accountthe phase transition. The solvability of the mathematical model and the convergence to the limit in terms of relaxation time, were studied. In the limit case, it is proved that the original problem is a Stefan-type

problem. Existence and uniqueness of a solution of one model of Stefan-type filtration theory with phase relaxation was obtained. The lemma of convergence to the limit in terms of relaxation time is proved. Numerical experiments are presented.

The third section of dissertation is devoted to study of the solvability of problems for a pseudoparabolic equation with linear boundary condition. The existence of weak solution of the problem considered here, is studied using Galerkin approximation and some a priori estimates. To prove the uniqueness of the weak solution of the problem, the Sobolev embedding theorem and the Gronwall-Bellman lemma are used. Along with this, the blow-up of solution of the problem in finite time is proved. The global solvability of the initial-boundary value problem and the uniqueness of a weak generalized solution are studied. The end of section deals with behavior of the solution in time.

The fourth section is devoted to the fundamental problem of solvability of initial boundary value problems for a quasilinear parabolic equation. The existence of weak solution of the inverse problem is proved by Galerkin method. Sufficient conditions for the finite-time decay of the solution in the measured region with a homogeneous Dirichlet condition were obtained, and also the stability of solution of nonlinear power-type opposite-sign inverse problem was obtained.

The fifth section of the dissertation is devoted to a competition system of two PDEs with nonlinear cross-diffusion. One shows that for some range of parameters there are two different types of periodic stationary solutions. Using them, partition the eight-dimensional parameter space shows Turing regions where the solutions exist.

The presentation reflects the applicant's goals, objectives . The sections are logically connected; as a whole, it has an internal unit.

The applicant from June to September 2018 completed a scientific internship at the University of Pécs. During her stay at university Zhanuzakova D.T. actively worked on her dissertation, consulted with best scientists of the Faculty.

The results of her research were published in 3 papers in high-ranking international journals (related to the international scientometric databases Web of Science and Scopus).

While working on her dissertation, Zhanuzakova D.T. showed all the qualities of a good researcher - ability to learn and understand new materials quickly, diligence, has new ideas, works concentrated with responsibility.

The Thesis is of high scientific level and meets the requirements for PhD dissertations, and its author **Zhanuzakova Dinara T. deserves the Doctor of Philosophy (PhD) degree in 6D060100 - Mathematics**.

Robert Kersner

Robert Kersner
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